An augmented canonical gravity wave

Daniel Strano

4 Ursula Court, Mendham, NJ 07945, US

E-mail: `stranoj@gmail.com`

August 2017

Abstract. Mechanical paths of the Einstein-Hilbert action that vary the action from its minimum are physically allowable by Feynman path integral quantization. Motivated by quantizing the action, we add trial monopole interactions to the Schwarzschild and Reissner-Nordström metrics, carried by a massless particle. The action was analyzed using Wolfram Mathematica and exact expressions, without numerics or arbitrary precision numbers. We find that the Schwarzschild and Reissner-Nordström solutions are degenerate as vacuum solutions in every linear combination of these radiating wave components. These solutions imply charged and uncharged graviton multipoles, as well as spin-aligned, massless charged graviton dipole production from four or more coincident, spin-aligned photons, such as could be produced via stimulated emission in a tuned laser. Uncharged gravitons should also be commonly bound in multipoles, whether charged gravitons are barred.

Acknowledgments Many thanks go to Allan Hungria of the math department at the University of Delaware, for help with fundamental mathematical considerations, to Peter Schmitt of S&A Technologies, for his essential feedback, and to Kim Ferris, for his guidance in the review process.

1. Introduction

Applying the canonical quantum theory to the canonical modern theory of gravity exactly, with no or few new axioms, is regarded as extremely difficult. Quantizing the gravitional field leads to nonrenormalizable infinities that preclude a computationally useful physical description of gravity. "Canonical quantum gravity" proper follows the Hamiltonian formulation of mechanics. It was established decades ago [2][9], and Hamiltonian approaches based on this early work are still actively researched today [23]. Feynman path integrals applied to general relativity have led to useful results, as well [14]. A minisuperspace approximation scheme can give a normalizable canonical result in quantizing black holes, despite the difficulties with quantizing the field directly[3]. However, "the quantum theory of gravity" is perceived as incomplete or unsatisfactory.

Experiment confirms general relativity's predictions of redshift and deflection of light by mass [25], and the existence of gravitational waves [1]. While general relativity and gravitational waves have general experimental support, the canonical mathematical form for gravity waves has proved quantum mechanically problematic. This is often taken as motivation to modify quantum or general relativistic theory [4] [17] [23]. However, there is broad consistency between prediction and experiment in application of both these theories to all phenomena but each other.

Gravity waves coupled to the mass quadrupole have been detected and undoubtedly exist. We expect all single gravitons to carry 2 units of spin, but interactions of other rank could be mediated by composite graviton particles, or multipoles. The quantum mechanical likelihood of gravitational interactions of other ranks should be determinable from how far the Einstein-Hilbert action values of these interactions deviate from classical solutions. However, any modification to the mathematical form of gravity waves must be able to explain observed modes of gravitational oscillation to date. Augmenting the wave equation, we might avoid modifying both the most bedrock principles of general relativity and quantum mechanics, to address the obvious physical intractabilities of quantum gravity. By restricting to real, rather than virtual, gravitons, we hope to avoid nonrenormalizability in a limited treatment of gravitons.

2. Method

2.1. Theory

The Euler-Lagrange equation selects the minima or extrema of the corresponding action, according to the principle of least action. The Einstein field equations are the solution of the Euler-Lagrange tensor equation for the Einstein-Hilbert action. The action depends only on the curvature for a vacuum solution:

$$S(n,m) = \int_{t_n}^{t_m} \int_V \frac{1}{16\pi G_N} R\sqrt{-g}\, d^3x\, dt, \tag{1}$$

[11], where R is the Ricci scalar, $\sqrt{-g}$ is the volume element, and G_N is Newton's constant. For non-vacuum solutions, a Lagrangian density for any non-gravitational matter fields is added to the integrand. The Einstein field equations are produced by solving the Euler-Lagrange equations or, equivalently, extremizing the action.

The volume integral is usually taken over the entire space, if possible. The boundary conditions of the Einstein-Hilbert action must be accounted to give a meaningful mechanical description via the principle of least action. Consider the variation of the Einstein-Hilbert action:

$$\delta A_{EH} = \int_V d^4x\sqrt{-g}\left(G_{\mu\nu}\delta g^{\mu\nu}\right) + \int_V d^4x\sqrt{-g}\nabla_\epsilon\left(g^{\mu\nu}\delta\Gamma^{\epsilon}{}_{\mu\nu} - g^{\epsilon\eta}\delta\Gamma^{\mu}{}_{\mu\eta}\right) \tag{2}$$

where $G_{\mu\nu}$ is the inverse Einstein tensor, $\Gamma^{\epsilon}{}_{\mu\nu}$ are the Christoffel symbols, $g^{\mu\nu}$ is the inverse of the metric tensor, and g is the metric tensor trace [20]. Over a finite volume, we must include the value at its boundary. Some research suggests this boundary and its interior bulk are holographically dual [19]. Rewritten from equation 2, this term is

$$\delta A_{boundary} = \int_v d^3x\sqrt{h}\left(Kh^{ij} - K^{ij}\right)\delta h^{ij}, \tag{3}$$

where h^{ij} is the inverse induced 3-space metric, h is its trace, and K^{ij} is the inverse extrinsic curvature [20].

The Feynman path integral formulation of quantum mechanics is a Hamilton-Jacobi approach that allows us to quantize by assigning phases to mechanical paths by their classical actions:

$$\int d\vec{x}_{N-1} \int d\vec{x}_{N-2} ... \int d\vec{x}_2 \prod_{n=2}^{N} \exp\left(\frac{iS(n, n-1)}{\hbar}\right) \tag{4}$$

is a solution to the Schrödinger equation (for a nonrelativistic action) in the limit where the time step goes to zero, where $\int d\,\vec{x_n}$ usually represents a an integral over the position basis at time step n [24]. We can think of this position basis integral as rather an integral over a general configuration basis over all space at a time step.

From equation 4, virtually any arbitrary variation in the action appears to have quantum mechanical meaning and quantifiable probability. However, adding any arbitrary hypothetical charge, wave interaction, negative energy, or similar supposition to the action might lead to a vacuum solution while not having a corresponding physical principle to mediate it. For example, in the action of the Schwarzschild solution, we should be able to arbitrarily vary the apparent Schwarzschild radius locally, which leads to wave propagation *if a physical particle exists that can mediate this particular wave.* Taking a time coordinate t' that is retarded by the speed of gravity or light,

$$r_s \rightarrow \not{r}_s = r_s + \not{r}_g = r_s(t') + 2\int_0^{r_s/2} b(k, t') \sin(kt')\, dk \tag{5}$$

such a quantum mechanical monopole variation is physical if a mediator scalar particle exists, where $b(k, t')$ is a general, arbitrary wave amplitude function, made proportional to the energy of the black hole. For such a particle to exist, contributing only to the Ricci scalar and not the matter field Lagrangian, it must be a composite particle of gravitons. Reports of similar monopole generalizations of general relativity, with a quantization condition, already exist in the literature [7].

To extend to the Reissner-Nordström metric, we add an analagous variation in the charge monopole:

$$q \rightarrow \not{q} = q + \not{q}_g = q(t') + \int_0^{q} a(k, t') \sin(kt')\, dk. \tag{6}$$

Our analysis was restricted to Reissner-Nordström with an exactly extremal ratio of charge to mass. No specific restraint on charge-to-mass ratio of gravitons might be required, though the author analyzed the case of emitted gravitons that maintain the exactly extremal limit, for reasons we will elaborate.

Starting from a variation of the local apparent Schwarzschild radius, we further assume that the underlying quantum mechanical basis waves travel at the speed of light. We also assume these waves carry the energy of a massless particle away from the black hole:

$$\frac{dM}{dt'} = \int_0^{\infty} \hbar k \frac{\partial b(k, t')}{\partial t'}\, dk, \tag{7}$$

where M is the mass of the black hole. This constrains the parameterization of r_s:

$$\frac{dr_s}{dt'} = -2G_N \frac{dE}{dt'}. \tag{8}$$

Charge is carried away from black holes analogously. The underlying particles must be emitted in a way that also preserves momentum, such as pairwise emission of equal energy particles in opposite directions. They should also carry 2 units of spin.

No gravitational effect in general relativity can propagate faster than the speed of light [5], so assume our gravitational quantum variations propagate along null geodesics. From the Schwarzschild line element, allowing $r_s \to \mathfrak{r}_s$, we can define t' (recursively) as an exterior, outgoing retarded time coordinate which is zero on the event horizon at $t = 0$. The coordinates are more easily defined implicitly via their inverse map:

$$t = \frac{1}{2}\left[t' + r' + W\left(\frac{1}{e}\right)\right], \tag{9}$$

where W is the Lambert-W function, and

$$r = \mathfrak{r}_s(t')\left(W\left\{\frac{1}{\mathfrak{r}_s(t')}\exp\left[\frac{r'}{2\mathfrak{r}_s(t')} - \frac{t'}{2\mathfrak{r}_s(t')} - 1\right]\right\} + 1\right) - W\left(\frac{1}{e}\right) \tag{10}$$

Note that the dependence of $\mathfrak{r}_s$ on t' is an arbitrary parameterization, which is important when inverting the coordinate transformation.

Reissner-Nordström has a different pair of coordinates, which can only be inverted in terms of an uncommon, but "well-behaved" transcendental function related to the Lambert-W. By trial and error, the author has found that one should take $t' = 0$ at $r = r_s$ and $t = 0$. In spherical coordinates, there is a simple replacement for t:

$$t = r' - t'. \tag{11}$$

Suppressing the argument of $\mathfrak{r}_s(t')$ for brevity, r can then be defined implicitly as a solution to the equation

$$\frac{2r^2 - 2r(r' + \mathfrak{r}_s - 2t') + \mathfrak{r}_s(2r - \mathfrak{r}_s)[\log(2r - \mathfrak{r}_s) - \log(\mathfrak{r}_s)] + \mathfrak{r}_s(r' - 2t')}{2r - \mathfrak{r}_s} = 0. \tag{12}$$

With Mathematica, one can easily define a custom transcendental function which solves this equation given specific arguments, and this is sufficient to carry out our analysis.

This is enough to form our actions in coordinates that mix t and r with t' and r'. Then, we can use the Jacobian to transform coordinates and derivatives to write the action entirely in terms of t' and r'. We need to check the boundary conditions of the action for validity. We avoid all arbitrary precision numerics.

2.2. Computational simplification

All computational analysis was carried out in Wolfram Mathematica. Several notebooks demonstrating the results are available along with this paper. A publicly available differential geometry package [15] was used to form the Ricci scalar and volume element for the Einstein-Hilbert action. The same package was used to form the extrinsic curvature tensor, the extrinsic curvature trace, and induced metric in order to check the boundary conditions. The resulting Mathematica notebook files are highly optimized and run to completion in minutes on a home computer.

The Mathematica function "Hold" and functional "Inactive" were used to reduce computational overhead. "Hold[x]" is equivalent to "x" when used in a Mathematica expression, but the form of its argument is "held" without evaluation. "Inactive[f]" prevents Mathematica from attempting to apply a function f, leaving it present with all its usual properties, but never attempting to evaluate an "Inactive" function. (An earlier attempt at reporting results for this method, by the same author, reported an incorrect action due to the use of "Hold" while entering the function as an "excluded form" in Mathematica's "Simplify" and "FullSimplify" functions. A support ticket was opened with Wolfram, but the author removed use of "Hold" as an excluded form, or any expression as an excluded form, for "Simplify" and "FullSimplify.")

The goal is an action entirely in terms t' and r' rather than t and r. Mathematica finds coordinate transformations between t and r, and t' and r', with the use of the "ProductLog" function, or Lambert W function, such that $ProductLog[z]$ can be defined as the solution of $z = we^w$ for w. The partial derivatives for the Jacobian were formed with this transformation. The boundary conditions were checked for both a t hypersurface and a t' hypersurface, and provide no contribution to the action. We form the Schwarzschild metric in the usual (t, r, θ, ϕ) basis, but we allow function dependence on t' and r'. Explicit dependence of t' and r' on t and r was carried through in all cases for Mathematica to recognize the need for t and r derivatives, until t and r derivatives could be substituted out of an expression. Without explicit dependence, $\frac{\partial t'}{\partial t}$ and $\frac{\partial t'}{\partial r}$ would be dropped incorrectly from expressions. Using equations 7 and 8, derivatives of $r_s(t')$ were systematically substituted for their equivalent in emitted wave amplitude. At each step, the notebooks programmatically check the partially transformed action for presence of $r_s(t')$ derivatives. After each step these derivatives are found, they are immediately removed by this same substitution. Before the explicit dependence of "$t'(t,r)$' and "$r'(t,r)$"' on t and r is removed, it is programmatically checked that the expression contains no derivatives of t' or r'. The explicit dependence is then removed, and t r are directly substituted out entirely in terms of t' and r'.

Mathematica is not directly capable of solving for a closed form for t' and r' derivatives with an exact definition of these coordinates, with "$Solve$[...]" or "$Reduce$[...]". The author's approach was to replace dependence on $\not{r}_s(t')$ with dependence on $r_s(t')$ in the definitions of t' and r'.This approximation should reproduce the average behavior over full gravity wave wavelengths, since the average monopole contribution over full wavelengths is zero, since it is the average over full sinusoidal waves. It was checked that the boundary conditions give zero contribution for either the exact definition or our approximation. However, we are restricted to integration over full monopole term wavelengths, after this approximation.

The expression is then out of mixed coordinates, but unsimplified and extremely unwieldy. Direct simplification by built-in Mathematica functions takes an extremely long time. Hence, linear superposed wave components are entered as test forms for

$b(k,t')$. "Dispatch" was used to substitute function arguments to help simplify the expression with good computational performance. Up to this point in the program, no numerical functions or arbitrary precision math is used, using effectively "losslesss" Mathematica operations. This relies entirely on exact numbers and symbols, by Mathematica's standard of exact numbers, and does not suffer from loss of precision due to "machine epsilon" or float rounding.

3. Results

Analysis with Mathematica shows that every linear combination of variation wave components is a vacuum solution to the Einstein field equations. (Remember that our motivation was to find the degree to which the monopole interactions deviate from the path of least action.) Waves emitted by Schwarzschild carry net mass away from the black hole, while waves emitted by an extremal Reissner-Nordström black hole carry energy and charge. Extending the treatment to a more general subset of a (Lorentz boosted) Kerr-Newmann solution is computationally challenging, but we can develop particle mechanics and thermodynamic considerations on the basis of Reissner-Nordström and Schwarzschild.

4. Discussion

4.1. Particle mechanics

Birkhoff's theorem implies that a stationary, static black hole cannot emit gravity waves. This can follow, in part, from the argument that the R_{tr} component of the Ricci tensor must be time-independent for a vacuum solution, and that the radial component of the metric must therefore also be time-independent. Our hypothetical gravity wave manifestly does not satisfy the assumptions of this argument, since our wave is a scalar which couples to the monopole. (Also, we only know it to be vacuum on average over full wavelengths.)

This augmented wave approximately obeys a law analagous to Gauss' Law,

$$\Phi_G = \frac{M(t')}{4\pi G}\left(1 + \int_0^{\frac{M(t')}{2\pi}} b(k,t')\sin(kt')\,dk\right) = \oiint_S \boldsymbol{G}\cdot d\boldsymbol{A}, \tag{13}$$

where the wave term is quantum mechanically equiprobable in all energy-conserving b. Again, the Einstein-Hilbert action for a scalar interaction was constructed to test the deviation from the least action, but we find that the action for this variation is vacuum. This augmentation of the metric leads to a general family of radiating vacuum solutions to the Einstein field equations. However, it is possible that no physical particle exists to mediate this augmentation, in which case their nature as vacuum solutions is moot.

The Reissner-Nordström solution motivates charged gravitons. If a charged graviton can exist, our treatment implies that we should not observe it with hyperextremal charge relative energy, $E^2 < q^2$, as this seems necessary to preserve Penrose's cosmic censorship hypothesis [21] when the particle is absorbed by a black hole. As these gravitons are not fundamentally confined, but confined only up to a high temperature, fundamental charge values of ± 1 and 0 are likely to be allowed.

Let us assume, for all further discussion, that a point-like model of the graviton holds to (sufficient) trans-Planckian scales. Then, if a thermal gas of gravitons emitted from a black hole contains many positively and negatively charged gravitons, they would tend to be bound in zero net charge multipoles of small moment. (Further, if oppositely charged gravitons are antiparticles, we expect annihilation of bound pairs, but ignore this momentarily.) Assuming a point-like model, the de Broglie wavelengths of low energy gravitons imply that nearly direct collision, leading to binding, is unlikely in a gas of free gravitons, but the direct production of bound graviton multipoles would still be possible.

Charged dipoles would oscillate to a distance of about one Planck length at a temperature of u^2 or α, the fine structure constant (in Planck units), at which point they are effectively freed at the scale of the gravitational interaction. This confines them below extremal ratios of energy to charge, below α times the Planck energy. (This is roughly on order of or higher than commonly expected grand unification energy scales for the other three fundamental forces, at about $9 \times 10^{25} eV$.) To have net magnetic and electric fields close to zero, with zero net spin, configurations would have to be bound in quadrupoles of two positive and two negative charges. The charges and electromagnetic fields would completely cancel in the limit of zero kinetic energy and zero graviton separation, which seems to be the absolute gravito-electromagnetic vacuum state of our field. The oscillating multipoles radiate electromagnetically, but the momentum carried must come from the original graviton multipole. The multipole and radiation from it would travel about the same direction at the speed of light, so emitted photons can be reabsorbed by the multipole. This would result in oscillation between gravitational kinetic energy and electromagnetic potential energy, with little or no effective net radiation perpendicular to the path of the multipole.

Our radiating vacuum solutions also allow entangled, pair-wise, opposite emission of charged gravitons, but only if these gravitons have enough kinetic energy to overcome their attraction to each other. This requires the "confinement energy" of α if we suppose that the gravitons "originated at the singularity" at some arbitrary time in the past and attracted as they passed out of the electrogravitational well. (For our Reissner-Nordström augmentation to exist and carry charge, though, it should again take the form of a scalar, in a multipole with cancelling overall spin.)

If a charged graviton with energy greater than confinement scale is absorbed by a black hole, freed from bound partners, the charged graviton necessarily carries the energy sufficient to keep the black hole at or above an extremal ratio to charge. Even for a hyperextremal charged graviton, the requisite energy for a charged graviton approaching from infinity to overcome the electrostatic repulsion of an extremally charged black hole of same charge sign is still the minimum implied by this bound. (This suggests an analogous screening by deflection of a hyperextremal spinning graviton due to "frame-dragging" by an extremal black hole with the same axis of spin.) The modification to gravity due to high electromagnetic fields and at short distances is then subtle rather gross, due to the possibility of a charged graviton, or any charged, massless, boson. However, subextremal charged graviton effects could contribute to the force at very short range, i.e. Planck scale (or α scale) effects.

Our Gauss' Law analog gives the appearance of gravitational standing waves (travelling at c) with a node on the event horizon of the black hole it exits from. Graviton multipoles can afford us this standing wave behavior, as potential gravitational and electromagnetic energy is constantly exchanged with the kinetic energy of the gravitons, causing the apparent gravitational field to oscillate in strength. The constant term, which represents the canonical form assumed for the gravity wave, must then still be carried by a chargeless graviton. However, a scalar multipole can be furnished by bound uncharged gravitons as well, if oppositely charged gravitons are annihilating antiparticles.

The momentum of a bound, uncharged graviton dipole could be reduced by the emission of additional gravitons from the dipole. As argued above, the emission of additional gravitons must occur with narrow spread around the original direction of propagation of the dipole, tending to be reabsorbed. We can avoid the intractabilities of virtual graviton fields, while naturally encompassing the nonlinear self-interaction of gravity, with a model in terms of real gravitons bound in generalized multipoles, which emit other gravitons, and can add and cancel (conserved) spin angular momentum to mediate any gravitational rank $2n$ interaction for n being an integer 0 or greater. (The idea of gravitational dipoles has been developed by others [13].)

Transforming to the center of mass frame of a multipole, the net momentum of the multipole is entirely transformed away, with the "poles" traveling toward or away from each other out to the separation where their kinetic energy becomes zero and the energy is entirely "potential," existing as other gravitons emitted from the poles, with zero net momentum and some fixed net angular momentum. If the original "poles" lack the kinetic energy to escape binding, so do all "potential" gravitons emitted by the separation of the "poles." Hence, the emitted field itself is bound, while the "original" poles may still lose and gain (conserved) kinetic and potential energy via exchange with other real, bound gravitons. While we argue total binding in the center of mass frame, real boson number is not conserved under the Unruh effect in an accelerated frame [8], so we might not conclude that total binding occurs in all frames.

The analagous radiation for the Kerr metric must carry net angular momentum. Hence, this radiation cannot be in the form of a scalar, with zero spin, and it cannot couple to the mass monopole term. While there is an internal coupling to the monopole, the mediating scalar must travel through the Kerr ergosphere to be emitted into the external region. Objects in the ergosphere must corotate, which scalar particles cannot do. If the Penrose process [22] separates scalar multipole components, such that part is ejected and part is reabsorbed, the ejected component carries net angular momentum, and the coupling is ultimately to the angular momentum and kinetic energy, effectively first reducing these quantities rather than the rest mass of the black hole past the extremal point. Similarly, if electromagnetic forces separate scalar multipoles made of cancelling charged particles, so that part of the multipole may exit the internal Cauchy horizon of a Reissner-Nordström black hole, charges opposite the net charge of the hole are drawn in while charges like the net charge of the hole are forced out, so the component ejected should carry nonzero charge (and zero spin, such as in the form of a magnetic quadrupole of two like charges). Both conditions apply to the Kerr-Newman metric, describing a rotating, charged black hole.

We might expect oppositely charged particles to be antiparticles of each other, causing charged graviton multipoles to self-annihilate. Whether these particle varieties are "true antiparticles," they can only annihilate by producing other available particle sets satisfying all conservation laws. As the bound charges travel at the speed of light, but along world lines curved by electromagnetic binding forces, the multipoles carry less net momentum-per-energy than the photon. A scalar charged multipole entails the breakdown of at least four gravitons, in an overall charge- and spin-neutral set, into at least two photons of opposite polarization. The two photons travel at an angle, reducing the net momentum from $E = pc$. For oppositely charged graviton pairs with aligned spin, at least four photons are required to conserve all quantities, traveling out with a spread of angles to reduce the net momentum.

This monopole coupling should be observed generally in matter. For charge exchange, a scalar charge quadrupole must be separated into two opposite sets of like-charged scalar magnetic quadrupoles, entailing energies on "α scale." Only at these high energies, certain fundamental particles might become their antiparticles or attain higher than fundamental charge, though increasing charge requires overcoming electrostatic repulsion. We expect the background temperature of charged gravitons to be very low, and we would not expect to observe these interactions typically in nature at the current cosmological epoch. Rest mass exchange would be relatively more common. If the known masses of the Standard Model particles represent the particles' gravitational ground states, gravitational rest mass excitation might still not be typically detected in the lab, but the relatively low background temperature of scalar graviton multipoles, due to black holes and cosmological artifact, could impart additional mass on astronomical scales of matter distribution. Since massive Standard Model particles acquire their masses via interaction with a scalar field with a nonzero vacuum expectation value, due to spontaneous symmetry breaking [6] [10] [12] [16], the observed fundamental masses cannot be reduced without reducing the expectation value for this field, by increasing the energy of the field. Therefore, the observed fundamental masses should be the ground states of the gravitational mass monopole interaction.

4.2. Thermodynamics

Having all these equiprobable modes of (uncharged multipole) breakdown available to any black hole naively implies a “particle lifetime” of r_s Planck units of time, by Fermi’s golden rule, and an average loss of half its energy as gravitational radiation in the event of breakdown. Most of the gravitons emitted would take on order of r_s Planck times to be emitted, implying a roughly constant thermal spectrum for astronomical black holes. The average loss of mass, before any consideration of background temperature, would be half a Planck mass per Planck time. Other conserved quantities including momentum, angular momentum, and potentially charge, should be radiated proportional to their fraction of “extremalness,” with extremal black holes following a well-known constraint

$$m^2 \geq a^2 + q^2, \tag{14}$$

in the Kerr-Newman metric, with assumed Lorentz invariance, implying

$$E^2 \geq m^2 + p^2 + a^2 + q^2, \tag{15}$$

[18] with energy E, rest mass m, momentum p, angular momentum parameter a, and charge q, such that $a/(2r_s)$ units of angular momentum should be radiated in a Planck time, and so for all conserved quantities that the graviton may carry. This is independent of whether radiation can only be perpendicular to the event horizon, or if it can emit at any angle.

These “naive” breakdown considerations are probably not realistic. They imply up to half of the mass of a body like Sgr A*, millions of solar masses, being emissible in a single graviton. The de Broglie wavelength for any graviton with Planck energy or greater fits within its own Schwarzschild radius, and this is not a case we should expect to treat with Fermi’s golden rule and perturbation theory without additional considerations.

Gravitons with de Broglie wavelengths that fit inside their Schwarzschild radii should be black holes with exactly extremal amounts of momentum, and therefore naked singularities. This suggests they cannot satisfy Penrose’s cosmic censorship hypothesis. The emission of a black body spectrum from a black hole, as per Hawking, is expected to carry greater thermodynamic entropy than that lost from the black hole, but the emission of a single graviton heavy enough to be a black hole itself does not. If an original black hole were to break into a lighter black hole and such an extremal black hole graviton, heuristically, the event horizon area of the remnant added to the event horizon area implied by the Schwarzschild radius of the graviton is less than the area of the original black hole, so emission is no longer a thermodynamically favorable, spontaneous process at or before this energy. A more realistic approximate model assumes only thermodynamically favorable graviton emission usually happens, with an average emissive power of approximately $E_P/(2r_s)$, with “E_P” being the Planck energy. This is a correction in addition to Hawking radiation, offset by a background temperature for our waves.

In the event that a second black hole covers some solid angle of emission of a first black hole, the net power released by the two is some amount less than this maximum power, as the two absorb a fraction of each other's emission. Bringing two test black holes closer together, the net power emitted should be gradually reduced, most obviously in the case of effective partial or total overlap between the event horizons, where emission from the interior portion of one event horizon cannot escape the other exterior horizon. Drawing two test black holes from infinitely distant to the point of total overlap of event horizons, we expect a smooth reduction of the net emission from the implied maximum power to half of the maximum value.

Charge neutral, zero spin multipoles made of charged particles are also available for mass monopole radiation, if charged gravitons exist. However, these should have high tendency to decay into photons. This could multiply the total gravitational radiance by about a factor of 5/4, assuming radiation occurs in quadrupoles of two oppositely charged particles apiece, but this extra component should be observed almost entirely as photons some time after emission. The expected power of emission has an inverse proportionality of black hole surface area to temperature, like Hawking radiation. For an object the size of Sgr A*, the photon temperature from this mode of breakdown would be about $0.02K$. For a black hole of about 6.6 solar masses, the temperature would be approximately $440K$ at its surface, suggesting a potentially observable infrared correction to observation of V616 Mon and small black holes in general. However, the binary nature of the nearest systems suspected to contain black holes allows many consistent spectra models, so that general failure of models with this correction could rule out charged graviton multipoles, but good fit does not necessarily provide confirmation of their existence.

4.3. General experimental design

If charged gravitons exist, we expect pair production of dipoles under the right circumstances. Specifically, a charged graviton dipole with aligned spin should have nearly cancelling electric and magnetic fields. Four coincident, spin-aligned photons of low energy should be capable of producing a confined graviton dipole. Three aligned-spin photons could be provided by a laser, while the fourth unit of spin could be supplied by virtual exchange with a nucleus, similar to lepton pair production. (The graviton is presumed massless and should also not carry lepton number.)

A laser with tuned gain could increase the fraction of the photon population that is coincident with the total spin of three units or greater. Since the poles of the graviton multipole are separated by less than the Planck length, creation by the interaction of photons from a laser that are not position entangled at the same point is unlikely, though one additional spin-aligned photon must be supplied from a different direction, in order to reduce total momentum from $E = pc$. Ideally, a laser should have its entire emitted photon population in sets of three photons entangled by stimulated emission. A scalar multipole cannot be produced this way due to spin angular momentum conservation, but higher multipole moments should also contribute, so long as their electromagnetic fields are effectively externally screened by the Planck length.

There would be almost total electromagnetic screening, though such exactly coincident photons would couple to virtual graviton pairs with the coupling constant of elementary charge leptons. Therefore, the dipole is not likely to be directly detected, but the energy loss from its production could be. The charges would have higher tendency to separate in the presence of an eletric field directed parallel to the dipole, and perpendicular to the laser beam, such as could be applied by the presence of a nucleus, with which an additional aligned unit of spin must be exchanged. A graviton dipole could annihilate to produce photons again, as explained above. They should tend to recombine into four photons scattering with a spread of angles. Energy and momentum conserving breakdown products appear to be relatively degenerate, so photons produced by annihilation could be randomly polychromatic. The chance of collision by the fourth photon is higher at greater photon energies due to the reduced de Broglie wavelengths, though it should be possible to achieve pair production with lasers with photon energies less than the masses of lepton pairs. If massless charged gravitons exist, pair production should be possible to the limit of no exchange with a nucleus or applied field, with only four coincident spin-aligned photons, though not necessarily with great frequency.

At high energies, if electron-positron pair production cross section goes like

$$\sigma \propto Z^2 \log\left(k_0 - k_{crit}\right) \tag{16}$$

with Z being the charge of the nucleus, k_0 being the incident photon wavenumber, and k_{crit} being the critical wavenumber for electron-positron pair production, at minimum sufficient to provide the mass of two electrons, then graviton pair production should go like

$$\sigma \propto \left(\frac{Z}{2}\right)^2 \log\left(k_0\right). \tag{17}$$

We assume here that the entirity of the laser is in spin triplet sets of photons, and that the spins of the charged nucleons are random. With ideal laser population statistics, the production of electron-positron pairs limits to a factor of 4 greater than graviton pairs. It is possible to approach a graviton cross section approximately equal to electron-positron cross section if all laser photons come in spin-aligned triplets and if charged nucleon spins are aligned with these triplets. The laser could be passed through a polarizer, and the nucleons could be magnetized. We see that, except under strictly ideal conditions, graviton pair production is significantly less than electron-positron pair production at high energies. At low energies, the photoelectric effect and Compton scattering appear to usually dominate. Below the threshold of the photoelectric effect, the impinging photon de Broglie wavelengths are large, and the chance of interaction is therefore low. In general, there might be no regime, or a very limited regime, where graviton pair production is expected to both occur at detectable levels and be the dominant mode of interaction. Detection might require a combination of careful experimental tuning and precise subtraction of these background processes.

5. Conclusion

Our scalar gravity wave augmentation leads to an infinite family of linearly superposable radiating vacuum solutions for the Reissner-Nordström and Schwarzschild metrics. If a physical particle exists that can mediate this wave, the only plausible candidate is a graviton multipole. Our model implies that gravitons carry ± 1 and 0 fundamental units of charge. The model also suggests a net emmission on order of $E_P/(2r_s)$ from black holes, in addition to Hawking radition, before the background temperature of these standing gravity waves is considered. Our vacuum solutions imply electric and magnetic scalar quadrupoles, of anti-aligned spin 2 gravitons. Such charged gravitons could be produced as pairs of from four or more coincident, spin-aligned photons of any energy, such as could be produced via stimulated emission, though detection might require careful tuning and background subtraction. Barring the existence of charged gravitons, real, uncharged gravitons can still be modeled in terms of general multipoles.

References

[1] Benjamin P Abbott, Richard Abbott, TD Abbott, MR Abernathy, Fausto Acernese, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, et al. Observation of gravitational waves from a binary black hole merger. *Physical Review Letters*, 116(6):061102, 2016.

[2] Richard Arnowitt, Stanley Deser, and Charles W Misner. Republication of: The dynamics of general relativity. *General Relativity and Gravitation*, 40(9):1997–2027, 2008.

[3] Abhay Ashtekar and Martin Bojowald. Quantum geometry and the Schwarzschild singularity. *Classical and Quantum Gravity*, 23(2):391, 2005.

[4] Carl Brans and Robert H Dicke. Mach's principle and a relativistic theory of gravitation. *Physical Review*, 124(3):925, 1961.

[5] S Carlip. Aberration and the speed of gravity. *Physics Letters A*, 267(2):81–87, 2000.

[6] Serguei Chatrchyan, Vardan Khachatryan, Albert M Sirunyan, Armen Tumasyan, Wolfgang Adam, Ernest Aguilo, T Bergauer, M Dragicevic, J Erö, C Fabjan, et al. Observation of a new boson at a mass of 125 Gev with the CMS experiment at the LHC. *Physics Letters B*, 716(1):30–61, 2012.

[7] YM Cho. Theory of gravitational monopole. Technical report, 1990.

[8] Luis CB Crispino, Atsushi Higuchi, and George EA Matsas. The Unruh effect and its applications. *Reviews of Modern Physics*, 80(3):787, 2008.

[9] Paul AM Dirac. The theory of gravitation in Hamiltonian form. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 246, pages 333–343. The Royal Society, 1958.

[10] François Englert and Robert Brout. Broken symmetry and the mass of gauge vector mesons. *Physical Review Letters*, 13(9):321, 1964.

[11] Richard P. Feynman. *Feynman Lectures on Gravitation*. Addison-Wesley Publishing, 1995.

[12] Gerald S Guralnik, Carl R Hagen, and Thomas WB Kibble. Global conservation laws and massless particles. *Physical Review Letters*, 13(20):585, 1964.

[13] Dragan Slavkov Hajdukovic. Quantum vacuum and virtual gravitational dipoles: the solution to the dark energy problem? *Astrophysics and Space Science*, 339(1):1–5, 2012.

[14] Stephen W Hawking. The path-integral approach to quantum gravity. In *General relativity*. 1979.

[15] M Headrick. A Mathematica package for tensor algebra and calculus. http://goo.gl/UDso5q, 2013. Accessed: 2015-12-16.

[16] Peter W Higgs. Broken symmetries and the masses of gauge bosons. *Physical Review Letters*, 13(16):508, 1964.

[17] Rosario Martin and Enric Verdaguer. Stochastic semiclassical gravity. *Physical Review D*, 60(8):084008, 1999.

[18] P O Mazur. Proof of uniqueness of the Kerr-Newman black hole solution. *Journal of Physics A: Mathematical and General*, 15(10):3173, 1982.

[19] T Padmanabhan. Holographic gravity and the surface term in the Einstein-Hilbert action. *Brazilian Journal of Physics*, 35(2A):362–372, 2005.

[20] T Padmanabhan. A short note on the boundary term for the Hilbert action. *Modern Physics Letters A*, 29(08):1450037, 2014.

[21] Roger Penrose. The question of cosmic censorship. *Journal of Astrophysics and Astronomy*, 20(3):233–248, 1999.

[22] Roger Penrose and RM Floyd. Extraction of rotational energy from a black hole. *Nature*, 229(6):177–179, 1971.

[23] Carlo Rovelli. Loop quantum gravity. *Living Rev. Rel*, 1(1):41–135, 1998.

[24] Jun John Sakurai and Jim Napolitano. *Modern Quantum Mechanics*. Second edition.

[25] Clifford M Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(1):4, 2014.