

# An augmented canonical gravity wave

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**Abstract.** Motivated by Feynman path integral quantizing the action, we add trial monopole interactions to the Schwarzschild and Reissner-Nordström metrics, carried by a massless particle. The action was analyzed using Wolfram Mathematica and exact expressions, without numerics or arbitrary precision numbers. We find that the Schwarzschild and Reissner-Nordström solutions are degenerate as vacuum solutions in every linear combination of these radiating wave components. These solutions imply charged and uncharged graviton scalar monopole terms, mediated by entangled graviton groups with cancelling spin acting as composite scalars. The implied charged gravitons, though exotic, should be confined til slightly below Planck scale and typically have negligible electromagnetic effect. We develop general experimental considerations for a simple experimental test, to produce charged graviton pairs from four coincident spin-aligned photons, from a tuned laser and a nucleus. We develop cosmological considerations for these waves as an explanation for dark energy and dark matter, and we discuss their compatibility with existing astrophysical data.

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## 1. Introduction

Applying the canonical quantum theory to the canonical modern theory of gravity exactly, with no or few new axioms, is regarded as extremely difficult. Nonrenormalizability of the canonical gravitational field precludes a useful or accurate physical description. “Canonical quantum gravity” proper follows the Hamiltonian formulation of mechanics. It was established decades ago [5][15], and Hamiltonian approaches based on this early work are still actively researched today [30]. Feynman path integrals applied to general relativity have led to useful results, as well [19]. A minisuperspace approximation scheme lead to a canonical result in quantizing black holes, despite the difficulties with quantizing the field directly[6]. However, “the quantum theory of gravity” is perceived as incomplete or unsatisfactory.

Experiment confirms general relativity's predictions of redshift and deflection of light by mass [35], and the existence of gravitational waves [1]. While general relativity and gravitational waves have general experimental support, difficulties with directly quantizing the canonical gravitational field are often taken as motivation to modify quantum or general relativistic theory [9] [22] [30]. However, there is broad consistency between prediction and experiment in application of both these theories to all phenomena but each other.

Gravity waves coupled to the mass quadrupole have been detected and undoubtedly exist. We expect all single gravitons to carry 2 units of spin. Like the electromagnetic force, higher pole interaction terms should contribute, which happen to correspond with overall spin values in groups of bosons entangled together by stimulated emission. If multipole interaction terms correspond to total spin values of entangled groups due to stimulated emission, we don't expect monopole interaction terms for either the spin 1 photon or spin 2 graviton. However, either particle might be able to mediate a monopole interaction term if it is physically possible to produce entangled spin-anti-aligned groups, with 0 overall spin.

The quantum mechanical likelihood of gravitational interactions of other ranks should be determinable from how far the Einstein-Hilbert action of these interactions deviates from classical solutions. However, any modification to the mathematical form of gravity waves must be able to explain observed modes of gravitational oscillation to date. In the following treatment, we will develop evidence for the existence of entangled spin-anti-aligned scalar graviton groupings mediating a gravitational monopole interaction, as well as evidence for their production by black holes, by presenting vacuum black hole solutions to the Einstein field equations with a monopole interaction, showing the theoretical economy that follows from assuming simply the existence of such solutions, and discussing the agreement of this model with existing observational data. By allowing entangled spin-anti-aligned graviton groupings, carrying 0 total and mediating a monopole interaction, somewhat similar to photons produced in stimulated emission, we might avoid modifying both the most bedrock principles of general relativity and quantum mechanics, to address the obvious physical intractabilities of quantum gravity. By restricting to real, rather than virtual, gravitons, we hope to avoid nonrenormalizability in a limited treatment of the quantum gravitational field.

## 2. Method

### 2.1. Theory

The Euler-Lagrange equation selects the minima or extrema of the corresponding action, according to the principle of least action. The Einstein field equations are the solution of the Euler-Lagrange tensor equation for the Einstein-Hilbert action. The action depends only on the curvature for a vacuum solution:

$$S(n, m) = \int_{t_n}^{t_m} \int_V \frac{1}{16\pi G_N} R \sqrt{-g} d^3x dt, \quad (1)$$

[17], where  $R$  is the Ricci scalar,  $\sqrt{-g}$  is the volume element, and  $G_N$  is Newton's constant. For non-vacuum solutions, a Lagrangian density for any non-gravitational matter fields is added to the integrand. The Einstein field equations are produced by solving the Euler-Lagrange equations or, equivalently, extremizing the action.

The volume integral is usually taken over the entire space, if possible. The boundary conditions of the Einstein-Hilbert action must be accounted to give a meaningful mechanical description via the principle of least action. Consider the variation of the Einstein-Hilbert action:

$$\delta A_{EH} = \int_V d^4x \sqrt{-g} (G_{\mu\nu} \delta g^{\mu\nu}) + \int_V d^4x \sqrt{-g} \nabla_\epsilon (g^{\mu\nu} \delta \Gamma^\epsilon_{\mu\nu} - g^{\epsilon\eta} \delta \Gamma^\mu_{\mu\eta}) \quad (2)$$

where  $G_{\mu\nu}$  is the inverse Einstein tensor,  $\Gamma^\epsilon_{\mu\nu}$  are the Christoffel symbols,  $g^{\mu\nu}$  is the inverse of the metric tensor, and  $g$  is the metric tensor trace [27]. Over a finite volume, we must include the value at its boundary. Some research suggests this boundary and its interior bulk are holographically dual [26]. Rewritten from equation 2, this term is

$$\delta A_{boundary} = \int_v d^3x \sqrt{h} (K h^{ij} - K^{ij}) \delta h^{ij}, \quad (3)$$

where  $h^{ij}$  is the inverse induced 3-space metric,  $h$  is its trace, and  $K^{ij}$  is the inverse extrinsic curvature [27].

The Feynman path integral formulation of quantum mechanics is a Hamilton-Jacobi approach that allows us to quantize by assigning phases to mechanical paths by their classical actions:

$$\int d\vec{x}_{N-1} \int d\vec{x}_{N-2} \dots \int d\vec{x}_2 \prod_{n=2}^N \exp\left(\frac{iS(n, n-1)}{\hbar}\right) \quad (4)$$

is a solution to the Schrödinger equation (for a nonrelativistic action) in the limit where the time step goes to zero, where  $\int d\vec{x}_n$  usually represents an integral over the position basis at time step  $n$  [31]. We can think of this position basis integral as rather an integral over a general configuration basis over all space at a time step.

From equation 4, virtually any arbitrary variation in the action appears to have quantum mechanical meaning and quantifiable probability. However, adding any arbitrary hypothetical charge, wave interaction, negative energy, or similar supposition to the action might lead to a description of a system which does not have a corresponding physical principle to mediate it, like many argue for an exact solution due to Alcubierre [4] that appears to rely on absolute negative energy. For example, in the action of the Schwarzschild solution, we should be able to arbitrarily vary the apparent Schwarzschild radius locally, but this implies a gravitational scalar field coupling to the mass monopole, while the hypothetical graviton is likely rank 2. Taking a time coordinate  $t'$  that is retarded by the speed of gravity or light,

$$r_s \rightarrow \mathfrak{z}_s = r_s + \mathfrak{z}_g = r_s(t') + 2 \int_0^{r_s/2} b(k, t') \sin(kt') dk \quad (5)$$

such a quantum mechanical monopole variation is physical if a mediator scalar particle exists, where  $b(k, t')$  is a general, arbitrary wave amplitude function, made proportional

to the energy of the black hole. For such a particle to exist, contributing only to the Ricci scalar and not the matter field Lagrangian, it must be a composite particle of gravitons. Reports of similar monopole generalizations of general relativity, with a quantization condition, already exist in the literature [14].

To extend to the Reissner-Nordström metric, we add an analogous variation in the charge monopole:

$$q \rightarrow \mathcal{Q} = q + \mathcal{Q}_g = q(t') + \int_0^q a(k, t') \sin(kt') dk. \quad (6)$$

Our analysis was restricted to Reissner-Nordström with an exactly extremal ratio of charge to mass, at the limit of a Cauchy horizon at the same radius as the Schwarzschild horizon. No specific restraint on charge-to-mass ratio of gravitons might be required, though the author analyzed the case of emitted gravitons that maintain the exactly extremal limit, for reasons we will elaborate.

No gravitational effect in general relativity can propagate faster than the speed of light [11]. Starting from a variation of the local apparent Schwarzschild radius, we assume that the underlying quantum mechanical basis waves travel at the speed of light. These waves should therefore also carry the energy of a massless particle away from the black hole:

$$\frac{dM}{dt'} = \int_0^\infty \hbar k \frac{\partial b(k, t')}{\partial t'} dk, \quad (7)$$

where  $M$  is the mass of the black hole. This constrains the parameterization of  $r_s$ :

$$\frac{dr_s}{dt'} = -2G_N \frac{dE}{dt'}. \quad (8)$$

Charge is carried away from black holes analogously. The underlying particles must be emitted in a way that also preserves momentum, such as pairwise emission of equal energy particles in opposite directions. The underlying fundamental particles should also carry 2 units of spin, but it might be possible to emit even numbers of entangled gravitons with cancelling overall spin.

These gravitational quantum variations, assumed to be carried by a discrete, massless particle, propagate along null geodesics. From the Schwarzschild line element, allowing  $r_s \rightarrow z_s$ , we can define  $t'$  (recursively) as an exterior, outgoing retarded time coordinate which is zero on the event horizon at  $t = 0$ . The coordinates are more easily defined implicitly via the inverse coordinate transformation:

$$t = \frac{1}{2} \left[ t' + r' + W\left(\frac{1}{e}\right) \right], \quad (9)$$

where  $W$  is the Lambert-W function, and

$$r = z_s(t') \left( W \left\{ \frac{1}{z_s(t')} \exp \left[ \frac{r'}{2z_s(t')} - \frac{t'}{2z_s(t')} - 1 \right] \right\} + 1 \right) - W\left(\frac{1}{e}\right) \quad (10)$$

Note that the dependence of  $z_s$  on  $t'$  is an arbitrary parameterization, which is important when inverting the coordinate transformation.

Reissner-Nordström has a different pair of coordinates, which can only be inverted in terms of an uncommon, but “well-behaved” transcendental function related to the Lambert-W. By trial and error, the author has found that one should take  $t' = 0$  at  $r = r_s$  and  $t = 0$ , for natural expression. In spherical coordinates, there is a simple replacement for  $t$ :

$$t = r' - t'. \quad (11)$$

Suppressing the argument of  $\mathfrak{z}_s(t')$  for brevity,  $r$  can then be defined implicitly as a solution to the equation

$$\frac{2r^2 - 2r(r' + \mathfrak{z}_s - 2t') + \mathfrak{z}_s(2r - \mathfrak{z}_s)[\log(2r - \mathfrak{z}_s) - \log(\mathfrak{z}_s)] + \mathfrak{z}_s(r' - 2t')}{2r - \mathfrak{z}_s} = 0. \quad (12)$$

With Mathematica, one can easily define a custom transcendental function which solves this equation given specific arguments, and this is sufficient to carry out our analysis.

This is enough to form our actions in coordinates that mix  $t$  and  $r$  with  $t'$  and  $r'$ . Then, we can use the Jacobian to transform coordinates and derivatives to write the action entirely in terms of  $t'$  and  $r'$ . We need to check the boundary conditions of the action for validity. We avoid all arbitrary precision numerics.

## 2.2. Computational simplification

All computational analysis was carried out in Wolfram Mathematica. Several notebooks demonstrating the results are available along with this paper. A publicly available differential geometry package [20] was used to form the Ricci scalar and volume element for the Einstein-Hilbert action. The same package was used to form the extrinsic curvature tensor, the extrinsic curvature trace, and induced metric in order to check the boundary conditions. The resulting Mathematica notebook files are highly optimized and run to completion in minutes on a home computer.

The Mathematica function “*Hold*” and functional “*Inactive*” were used to reduce computational overhead. “*Hold[x]*” is equivalent to “ $x$ ” when used in a Mathematica expression, but the form of its argument is “held” without evaluation. “*Inactive[f]*” prevents Mathematica from attempting to apply a function  $f$ , leaving it present with all its usual properties, but never attempting to evaluate an “*Inactive*” function. (An earlier attempt at reporting results for this method, by the same author, reported an incorrect action due to the use of “*Hold*” while entering the function as an “excluded form” in Mathematica’s “*Simplify*” and “*FullSimplify*” functions. A support ticket was opened with Wolfram, but the author removed use of “*Hold*” as an excluded form, or any expression as an excluded form, for “*Simplify*” and “*FullSimplify*.”)

The goal is an action entirely in terms  $t'$  and  $r'$  rather than  $t$  and  $r$ . Mathematica finds coordinate transformations between  $t$  and  $r$ , and  $t'$  and  $r'$ , with the use of the “*ProductLog*” function, or Lambert W function, such that  $ProductLog[z]$  can be defined as the solution of  $z = we^w$  for  $w$ . The partial derivatives for the Jacobian were formed with this transformation. The boundary conditions were checked for both a  $t$  hypersurface and a  $t'$  hypersurface, and provide no contribution to the action. We form

the Schwarzschild metric in the usual  $(t, r, \theta, \phi)$  basis, but we allow function dependence on  $t'$  and  $r'$ . Explicit dependence of  $t'$  and  $r'$  on  $t$  and  $r$  was carried through in all cases for Mathematica to recognize the need for  $t$  and  $r$  derivatives, until  $t$  and  $r$  derivatives could be substituted out of an expression. Without explicit dependence,  $\frac{\partial t'}{\partial t}$  and  $\frac{\partial r'}{\partial r}$  would be dropped incorrectly from expressions. Using equations 7 and 8, derivatives of  $r_s(t')$  were systematically substituted for their equivalent in emitted wave amplitude. At each step, the notebooks programmatically check the partially transformed action for presence of  $r_s(t')$  derivatives. After each step these derivatives are found, they are immediately removed by this same substitution. Before the explicit dependence of “ $t'(t, r)$ ” and “ $r'(t, r)$ ” on  $t$  and  $r$  is removed, it is programmatically checked that the expression contains no derivatives of  $t'$  or  $r'$ . The explicit dependence is then removed, and  $t$   $r$  are directly substituted out entirely in terms of  $t'$  and  $r'$ .

Mathematica is not directly capable of solving for a closed form for  $t'$  and  $r'$  derivatives with an exact definition of these coordinates, with “*Solve[...]*” or “*Reduce[...]*”. The author’s approach was to replace dependence on  $z_s(t')$  with dependence on  $r_s(t')$  in the definitions of  $t'$  and  $r'$ . This approximation should reproduce the average behavior over full gravity wave wavelengths, since the average monopole contribution over full wavelengths is zero, since it is the average over full sinusoidal waves. It was checked that the boundary conditions give zero contribution for either the exact definition or our approximation. However, we are restricted to integration over full monopole term wavelengths, after this approximation.

The expression is then out of mixed coordinates, but unsimplified and extremely unwieldy. Direct simplification by built-in Mathematica functions takes an extremely long time. Hence, linear superposed wave components are entered as test forms for  $b(k, t')$ . “*Dispatch*” was used to substitute function arguments to help simplify the expression with good computational performance. Up to this point in the program, no numerical functions or arbitrary precision math is used, using effectively “lossless” Mathematica operations. This relies entirely on exact numbers and symbols, by Mathematica’s standard of exact numbers, and does not suffer from loss of precision due to “machine epsilon” or float rounding.

### 3. Results

Analysis with Mathematica shows that every linear combination of variation wave components is a vacuum solution to the Einstein field equations. (Remember that our motivation was to find the degree to which the monopole interactions deviate from the path of least action. Also, remember that our test variation is not fully general and arbitrary at this point, such that we do not mean that a perfectly arbitrary variation always produces a solution.) Waves emitted by Schwarzschild carry net mass away from the black hole, while waves emitted by an extremal Reissner-Nordström black hole carry energy and charge. Extending the treatment to a more general subset of a (Lorentz boosted) Kerr-Newman solution is computationally challenging, but we can

develop particle mechanics and thermodynamic considerations based on the assumption that this generalization exists.

## 4. Discussion

### 4.1. General considerations

The reader might think that, if our motivation is Feynman path integral quantization, our variation should be entirely general and not restricted to these scalar waves. We stress that our analysis shows that these metrics all have exact vacuum action, when self consistent amplitudes and wavelengths are chosen. They are therefore (degenerate) exact vacuum solutions to the (nonquantum) Einstein field equations, as valid as Schwarzschild prior to explicit quantization. We derive these new nonquantum solutions to the Einstein field equations by assuming only the existence of an implicitly underlying discrete, massless gravitational force carrier particle, without treating it directly. At this point, we can completely dispense with Feynman path integrals and any form of explicit quantization for the composite system, if we wish, while still assuming the existence of an underlying discrete force carrier. However, these solutions' degeneracy is easily interpreted in terms of energy degenerate quantum eigenstates, assuming generalizability of our treatment to a Lorentz boosted Kerr-Newman metric. We might not as easily interpret this degeneracy in the Einstein field equations without knowledge of the resulting trivial Feynman path integral. We liken our solutions to Alcubierre's, because they are either physically realized, barred by the requirement of an exotic, nonphysical particle, or else fundamentally theoretically problematic for general relativity due to their degeneracy with the Schwarzschild solution. We submit an explicit derivation alongside this paper, in the form of Mathematica notebooks, and eagerly invite criticism of the method.

Note that all that follows depends simply on the existence of solutions for such massless, discrete, vacuum, scalar gravitational waves. Almost nothing we are about to develop depends on any other particular aspect of the solutions' form, except that solutions exist for black hole metrics with linearly superposable, vacuum, scalar monopole radiation that couples to all conserved quantities, as we assert that our proposed solutions are for mass and charge. We offer our computational derivations as explicit, rigorous, and faithful, but we appreciate that this could be difficult to confirm; the exact correctness of our proposed solutions' forms does not matter.

### 4.2. Particle mechanics

Birkhoff's theorem implies that a stationary, static black hole cannot emit gravity waves. Implicit in common derivations of Birkhoff's theorem is the assumption that a vacuum Ricci tensor leads to spherically symmetric metrics that must be independent of time [12]. However, it can be checked that our solution still results in a spherically symmetric vacuum Einstein-Hilbert action and vacuum Ricci tensor with a time dependent

metric. We satisfy the expected form for a qualified metric up to purely geometrical considerations, but our modified Schwarzschild and Reissner-Nordström metrics still carry time dependence, despite vacuum action and vacuum Ricci tensor. However, we need not argue purely from the purported counterexample of this author's own solutions.

There is precedent in the literature for reasonable hypothetical counterexample against this specific intermediate assumption of Birkhoff's theorem, that a spherically symmetric vacuum solution must have a time-independent metric. Tryon specifically stated that his proposal of "vacuum genesis," of the universe arising as a vacuum fluctuation from a null observable state, required that the universe kept the overall quantum numbers of the vacuum, therefore requiring homogeneity and isotropy [34]. This implies a spherically symmetric, time-dependent metric, which would be a counterexample to an intermediate assumption upon which Birkhoff's theorem relies, in common textbook derivations. Even if our universe did not arise as a vacuum fluctuation, the simple existence of a vacuum solution to the Einstein field equations as per Tryon's vacuum genesis would disprove an intermediate assumption of Birkhoff's theorem by counterexample, as we argue our monopole interaction solution also does. If such a solution requires a minimal quantum deviation from perfect spherical symmetry, it seems to require only a tiny deviation from it at most, suggesting that this spherical symmetry requirement of Birkhoff's theorem, leading to time-independence, might be ruined by even deviations from spherical symmetry due only to uncertainty principle. To physicists' intuition, if Birkhoff's theorem were to actually rely intermediately on specifically perfect symmetry, it is likely to be an effectively trivial result of little physical significance, due to the general instability of perfect symmetry in practice in physical reality [32].

Our augmented wave approximately obeys a law analagous to Gauss' Law,

$$\Phi_G = \frac{M(t')}{4\pi G} \left( 1 + \int_0^{\frac{M(t')}{2\pi}} b(k, t') \sin(kt') dk \right) = \oiint_S \mathbf{G} \cdot d\mathbf{A}, \quad (13)$$

where the wave term is quantum mechanically equiprobable in all energy-conserving  $b$ .

A modified Schwarzschild solution with a mass monopole interaction term implies a scalar particle, while there is excellent evidence to suggest that the fundamental graviton carries two units of spin, implying a quadrupole interaction. However, the fundamental photon carries one unit of spin under all cases, implying a dipole interaction, while it is uncontroversial and well known that the electromagnetic interaction has a quadrupole term and higher order terms. Before we deal with gravity, how does a particle with one unit of spin mediate a quadrupole interaction, implying two units of spin?

In stimulated emission, photons are emitted entangled and spin-aligned. Position-entangled groups of photons can carry  $n$  units of spin with  $n$  being an integer greater than or equal to 1. These available spin states correspond with exactly the dipole, quadrupole and higher order terms of the electromagnetic interaction, as if entangled, spin-aligned photons act as composite mediators of higher order interaction terms. We'd probably expect the same behavior from gravitons produced by stimulated emission,



carrying  $2n$  units of spin, with  $n$  being an integer greater than or equal to 1, allowing the gravitational interaction to mediate interactions of higher pole number. A scalar interaction might surprise us, implying an entangled spin-anti-aligned state. With valid monopole interaction solutions in hand, though, they are easily explained by exactly this configuration, of entangled spin-anti-aligned gravitons. We will further show that such a gravitational monopole interaction has a great theoretical economy and agrees with existing observational data, supporting the validity our proposed solutions.

Our modified Reissner-Nordström solution motivates charged gravitons. We admit that this is exotic, and it might be physically barred for other reasons despite the author's assertion of the existence of a such a solution to the Einstein field equations, as stated earlier. So long as it follows that gravity's typical direct interaction with electromagnetism is negligibly tiny, though, this might be an exciting prospect for carefully tuned direct experimental tests of gravity, so let us follow this line of reasoning through fully and credulously.

If a charged graviton can exist, our treatment implies that we should not observe it with hyperextremal charge relative energy,  $E^2 < q^2$ , as this seems necessary to preserve Penrose's cosmic censorship hypothesis [28] when the particle is absorbed by a black hole. This implies confinement of hypothetical charged gravitons. Fundamental charge values of  $\pm 1$  and 0 are implied by several considerations. Firstly, known gravity wave modes must be carried by an uncharged graviton. Additionally,  $\pm 1$  would allow entangled gravitons carrying a charge monopole interaction to interact with a charged lepton to produce an oppositely charged anti-lepton, if the graviton can carry other conserved quantities like lepton number, as well. (This treatment ultimately suggests to the author that the gravitational interaction is capable of carrying all conserved quantities.) A scalar carrying like charge would always or almost always be unable to overcome electrostatic repulsion to impart the charge it carries to another charged fundamental particle, while oppositely charged leptons would attract scalar charged graviton groupings.

Of course, we see no obvious experimental or observational evidence that the graviton can ever interact electromagnetically, but fully developed consideration of this hypothetical charged graviton actually implies confinement and very weak, sub-Planck scale electromagnetic interaction. If a thermal gas of gravitons emitted from a black hole contains many positively and negatively charged gravitons, they would tend to be bound in zero net charge multipoles of small moment. (Further, if oppositely charged gravitons are antiparticles, we expect annihilation of bound pairs, but ignore this momentarily.) Our modified Reissner-Nordström solution implies that direct production of entangled, charged gravitons acting as composite scalars. Charged graviton dipoles emitted together this way would oscillate to a distance of about one Planck length at a temperature of  $u^2$  or  $\alpha$ , the fine structure constant (in Planck units), at which point they would be effectively freed at the scale of the gravitational interaction. This confines them below extremal ratios of energy to charge, below  $\alpha$  times the Planck energy. This is roughly on order of or higher than commonly expected grand unification

energy scales for the other three fundamental forces, at about  $9 \times 10^{25} eV$ . To be emitted with little potential energy, with net magnetic and electric fields close to zero and with zero net spin, configurations would have to be bound in quadrupoles of two positive and two negative charges. The charges and electromagnetic fields would completely cancel in the limit of zero kinetic energy and zero graviton separation, which seems to be the absolute gravito-electromagnetic vacuum state of our field. If this is the true gravito-electromagnetic vacuum point of the field, this also suggests how nonzero charge can arise from vacuum without infinite self-energy. The oscillating multipoles radiate electromagnetically, but the momentum carried must come from the original graviton multipole. The multipole and radiation from it would travel the same direction at the speed of light, so emitted photons can be reabsorbed by the multipole, also traveling at the speed of light. This would result in oscillation between gravitational kinetic energy and electromagnetic potential energy, with little or no effective net radiation perpendicular to the path of the multipole. This picture suggests confined, effectively negligible electromagnetic graviton interaction until distances smaller than the Planck length. Further, if these particles are emitted in scalar pairs and can annihilate, we do expect an excess of photons as a breakdown product from black holes, but we expect almost no other obvious electromagnetic interaction until past grand unification scale. (We will further discuss exactly this expected photon excess in one of the next sections.)

If the occurrence of black holes with significant net charge is rare, we expect the emission of graviton groupings with net charge to be rare, as well. It can be easily checked that if like charge gravitons were emitted entangled with opposite spin, magnetic and gravitational attractive forces between particles at sub-Planck distance from each other would overcome electrostatic repulsion, allowing them to exist as scalar groupings. We expect them to carry equal or greater energy than the extremal bound on energy versus charge implied by Penrose's cosmic censorship hypothesis.

If a charged graviton with energy greater than confinement scale is absorbed by a black hole, freed from bound partners, the charged graviton necessarily carries the energy sufficient to keep the black hole at or above an extremal ratio of mass to charge. Even for a hyperextremal charged graviton, the requisite energy for a charged graviton approaching from infinity to overcome the electrostatic repulsion of an extremally charged black hole of same charge sign is still the minimum implied by this bound. (This suggests an analogous screening by deflection of a hyperextremal spinning graviton due to "frame-dragging" by an extremal black hole with the same axis of spin.) The modification to gravity due to high electromagnetic fields and at short distances is again subtle rather than gross, due to the possibility of a charged graviton, or any charged, massless, boson.

The analogous radiation for the Kerr metric must carry net angular momentum, from orbit or spin. While our scalar radiation should internally couple to the mass monopole in a rotating black hole, the mediating scalar must travel through the Kerr ergosphere to be emitted into the external region. Objects in the ergosphere must corotate and are driven to spin opposite the spin of the black hole. If the Penrose process [29]

separates entangled composite scalar components, such that part is ejected and part is reabsorbed, the ejected component carries net spin angular momentum, effectively coupling it as a spin-2 interaction to the angular momentum and kinetic energy, first reducing these quantities rather than the rest mass of the black hole past the extremal point implied by cosmic censorship. Similarly, if electromagnetic forces separate scalar multipoles made of cancelling charged particles, so that part of the multipole may exit the internal Cauchy horizon of a Reissner-Nordström black hole, charges opposite the net charge of the hole are drawn in while charges like the net charge of the hole are forced out, so the component ejected should carry nonzero charge (and zero spin, such as in the form of a magnetic quadrupole of two like charges). Both conditions apply to the Kerr-Newman metric, describing a rotating, charged black hole.

These gravitational monopole couplings to mass and charge should be observed generally in matter. We expect the background temperature of charged gravitons to be very low, and we would not expect to observe charge monopole interactions typically in nature at the current cosmological epoch, due to the just sub-Planck scale energy required to separate massless charged bosons past Planck length. Rest mass exchange would be relatively more common. If the known masses of the Standard Model particles represent the particles' gravitational ground states, gravitational rest mass excitation might still not be typically detected in the lab, but the relatively low background temperature of scalar graviton groupings, due to black holes and cosmological artifact, could impart additional mass on astronomical scales of matter distribution. Since massive Standard Model particles acquire their masses via interaction with a scalar field with a nonzero vacuum expectation value, due to spontaneous symmetry breaking [13] [16] [18] [21], the observed fundamental masses cannot be reduced without reducing the expectation value for this field, by increasing the energy of the field. Therefore, the observed fundamental masses should be the ground states of the gravitational mass monopole interaction.

### 4.3. Thermodynamics

Consider only the purported mass monopole interaction, for now. Having all these equiprobable modes of breakdown available to any black hole naively implies a “particle lifetime” of  $r_s$  Planck units of time, by Fermi's golden rule, and an average loss of half its energy as gravitational radiation in the event of breakdown. (We develop this “naive” approach to show why it is probably wrong.) Most of the gravitons emitted would take on order of  $r_s$  Planck times to be emitted, implying a roughly constant thermal spectrum for astronomical black holes. The average loss of mass, before any consideration of background temperature, would be half a Planck mass per Planck time. Other conserved quantities including momentum, angular momentum, and potentially charge, should be radiated proportional to their fraction of “extremalness,” with extremal black holes following a well-known constraint

$$m^2 \geq a^2 + q^2, \tag{14}$$

in the Kerr-Newman metric, with assumed Lorentz invariance, implying

$$E^2 \geq m^2 + p^2 + a^2 + q^2, \quad (15)$$

[24] with energy  $E$ , rest mass  $m$ , momentum  $p$ , angular momentum parameter  $a$ , and charge  $q$ , such that  $a/(2r_s)$  units of angular momentum should be radiated in a Planck time, and so for all conserved quantities that the graviton may carry. This is independent of whether radiation can only be perpendicular to the event horizon, or if it can emit at any angle.

These “naive” breakdown considerations are probably not realistic. They imply up to half of the mass of a body like Sgr A\*, millions of solar masses, being emissible in a single graviton. The de Broglie wavelength for any graviton with Planck energy or greater fits within its own Schwarzschild radius, and this is not a case we should expect to treat with Fermi’s golden rule and perturbation theory without additional considerations.

Gravitons with de Broglie wavelengths that fit inside their Schwarzschild radii should be black holes with extremal or excess amounts of momentum, and therefore naked singularities. This suggests they cannot satisfy Penrose’s cosmic censorship hypothesis. The emission of a black body spectrum from a black hole, as per Hawking, is expected to carry greater thermodynamic entropy than that lost from the black hole, but the emission of a single graviton heavy enough to be a black hole itself does not. If an original black hole were to break into a lighter black hole and such an extremal black hole graviton, heuristically, the event horizon area of the remnant added to the event horizon area implied by the Schwarzschild radius of the graviton is less than the area of the original black hole. Though thermal radiation adds an amount of entropy, this is offset by reduction in entropy due to the reduction of total event horizon area [8]. It is clear that the spontaneous split of one black hole into two is not a thermodynamically favorable process, at least when all breakdown products carry about a Planck mass or greater. A more realistic approximate model assumes only thermodynamically favorable graviton emission usually happens, with an average emissive power of approximately  $E_P/(2r_s)$ , with “ $E_P$ ” being the Planck energy. This is a correction in addition to Hawking radiation, offset by a background temperature for our waves.

In the event that a second black hole of equal mass covers some solid angle of emission of a first black hole, the net power released by the two is some amount less than this maximum power, as the two absorb a fraction of each other’s emission. Bringing two test black holes closer together, the net power emitted should be gradually reduced, most obviously in the case of effective partial or total overlap between the event horizons, where emission from the interior portion of one event horizon cannot escape the other exterior horizon. Drawing two test black holes from infinitely distant to the point of total overlap of event horizons, we expect a smooth reduction of the net emission from the implied maximum power to half of the maximum value.

Charge neutral, zero spin multipoles made of charged particles are also available for mass monopole radiation, if charged gravitons exist. However, these should have high

tendency to decay into photons. This could multiply the total gravitational radiance by about a factor of  $5/4$ , assuming radiation occurs in quadrupoles of two oppositely charged particles apiece, but this extra component should be observed almost entirely as photons some short time after emission. The expected power of emission has an inverse proportionality of black hole surface area to temperature, like Hawking radiation. For an object the size of Sgr A\*, the photon temperature from this mode of breakdown would be about  $0.02K$ . For a black hole of about 6.6 solar masses, the surface temperature would appear to be approximately  $440K$  to an observer at infinity, suggesting a potentially observable infrared correction to observation of V616 Mon and small black holes in general. The wavelength peak from this temperature would be about  $6.6\mu m$ . Munor and Mauerhan report excesses at  $4.5\mu m$  and  $8\mu m$  from three quiescent low-mass black hole candidates [25]. However, the complicated binary nature of the nearest systems suspected to contain black holes might allow many reasonable and consistent spectra models, and might not be capable of providing strong proof for charged gravitons.

#### *4.4. Cosmology, dark energy, and dark matter*

These monopole waves should have a cosmological background temperature, which would have frozen out at the beginning of the Grand Unified Epoch and spread similar to a photon gas since that time. To agree with observation, the background temperature today need either be negligible, or else take a form whose identity is not well understood, which we posit could be “dark energy.” To be confused for a true cosmological constant, the boson gas must be relatively weakly interacting, which it is, and the apparent energy density must stay close enough to effectively constant with the expansion of space. We propose this weakly interacting gas temperature could stay effectively constant with the expansion of space due to being at a critical point of a phase transition, buffered in temperature by the evaporation of primordial black holes. Further, this background temperature would impart additional mass over large ensembles of baryons, “dark matter,” due to its nature as a gas of scalar particles coupling to the mass monopole term of gravity.

We can reconcile this hypothesis with limits imposed by observations that have already been made. Most of our estimates here depend directly on a high precision measurement of dark energy density, for which data in the literature is limited, beyond the current limits of the Planck collaboration [2]. We take an estimate of about  $7 \times 10^{-30} g/cm^3$ , a commonly quoted value for this quantity, and we estimate quantities that follow from it to the first significant digit or order of magnitude.

If dark energy were purely this scalar background, it would have a temperature of very roughly  $40K$ . LIGO has set a limit on the maximum amount of stochastic gravity wave background in the  $10Hz$  to  $100kHz$  range at not greater than  $6.9 \times 10^{-6}$  times the critical density of the universe, with 95% confidence [23]. The quantum mechanical distribution of energies in a photon gas puts our energy density in this frequency range on about the  $10^{-30}$  scale of critical density fraction, far below LIGO’s limit.

To sufficiently buffer the temperature of the gas with a phase transition, through cosmological expansion at the current epoch, about  $(3x)10^{37}kg$  worth of black holes around one Planck mass in size must evaporate per second across the whole of the observable universe for constant energy density. If this rate of evaporation were constant throughout the age of the universe, and if the local age of the universe we observe is linearly interpolated between the Big Bang and present day, (which is an extremely rough first-order approximation, but probably representing a reasonable upper limit, due to the actual distance dependence of redshifts), it implies total mass evaporation on  $(5\times)10^{51}kg$  scale compared  $10^{53}kg$  scale for the estimated mass contained in the entire observable universe. This appears to be the tightest bound on our model, since it is known that light primordial black holes do not contribute a significant fraction to the present critical mass density [3] [7] [10] [33]. However, the mass fraction of light primordial black holes evaporated up to current day need not be so close to the present day mass fraction. If the present dark matter mass fraction of light primordial black holes is no greater than 2%, this is sufficient to maintain the apparent constancy of the dark energy background for over a billion of years into the future, by our model.

This weakly interacting, massless scalar boson background couples to the mass monopole term of gravity and should impart additional rest mass to particles it interacts with and excites. Assuming the background is in thermal equilibrium with baryons, the Maxwell-Boltzmann distribution should describe the distribution of additional masses due to excitation at a given temperature. It is interesting to consider the possibility of a two-tailed Maxwell-Boltzmann distribution, with a spread of masses both heavier and lighter than the fundamental baryon masses, both representing excitations above the Higgs vacuum. However, the sign of rest mass carried by these particles would be non-arbitrary, either positive or negative, up to some potential (nonspatial) hyper-rotation transformation. Hence, we assume a broken symmetry and take a single-tailed distribution, with positive rest mass excitations.

Assuming no available mode of breakdown besides into a single baryon plus scalar composite graviton radiation, this model far overestimates dark matter content, by a factor of about  $(4\times)10^2$ . (Incidentally, note that even a breakdown-free model at this temperature implies virtually no chance of even a single baryon in the observable universe attaining the Planck mass or greater to become a black hole, with approximately  $10^{80}$  baryons in the observable universe.) Of course, for high rest mass excitations, we expect other high-energy breakdown reactions. For example, a neutron with excited rest mass should have significant interaction with the Higgs field. To conserve all required quantities, we might expect breakdown via a Higgs scalar, an anti-Higgs scalar, and a (less) excited neutron. Baryons crossing the energy threshold for this breakdown should freeze out the tail of the Maxwell-Boltzmann distribution for the temperature as they decay, being left with basically random excitation energy below the threshold. If this particular decay is the lightest likely mode of breakdown, we still overestimate dark matter content by a factor of about  $(3\times)10$ . The less energetic the lightest significantly likely breakdown products are, the less dark matter we estimate.

The ideal excitation level for agreement with the amount of dark matter observed is a rest mass of about 8 neutron masses (in addition to the ground state fundamental rest mass of a baryon). This implies a lightest significantly likely breakdown of a proton with excited rest mass similar to the following:

$$H^+ \rightarrow \alpha + \bar{\alpha} + H^+. \quad (16)$$

This is a an excited “heavy” proton becoming a proton plus a helium-4 nucleus and an anti-helium-4 nucleus. Rather than an  $\alpha$  particle, it might be even more favorable to produce two nuclei with equal numbers of baryons and antibaryons, equal positive charge and negative charge, cancelling overall spin, and cancelling overall magnetic moments, which is feasible by rearrangement of the nucleons in the particles produced in the above reaction. These products in excess of the original proton or neutron would quickly annihilate to produce further breakdown products, including electromagnetic radiation.

The author regrets that we lack data to provide better than an order of magnitude, rough feasibility analysis of this mode for the moment. However, this analysis is qualitatively and even quantitatively insensitive to a factor of at least 10 in either direction times the estimate for dark energy, except for the required mass of light primordial black holes, which also gains or loses about a factor of 10, and except for the significant digit of the Kelvin scale temperature, which is unimportant to us here except as an intermediary quantity. Much about the usual treatments of a hypothetical scalar particle as dark energy particle or inflaton apply to the scalar presented here. In the limit of no low mass primordial black holes for a phase transition model, this particle still closely resembles other hypothetical dark energy candidate scalars, and could be expected to perform similarly. Further, this model has no tunable parameter, if the mass distribution of primordial black holes can be ascertained by observation or self-consistently fixed in simulation and if the proposed breakdown products can be shown to be a requirement from first principles. The model has the potential to explain both dark matter and dark energy entirely or virtually entirely, in addition to being a scalar inflaton candidate. Further, perhaps best of all, we argue that this requires no “new physics,” that it results from a solution to the canonical Einstein field equations directly under only the further assumption of the existence of a massless, discrete gravitational force carrier.

#### 4.5. *General experimental design*

If charged gravitons exist, we expect pair production of dipoles under the right circumstances. Specifically, a charged graviton dipole with aligned spin should have nearly cancelling electric and magnetic fields. Four coincident, spin-aligned photons of low energy should be capable of producing a confined graviton dipole. Three aligned-spin photons could be provided by a laser, while the fourth unit of spin could be supplied by virtual exchange with a nucleus, similar to lepton pair production.

A laser with tuned gain could increase the fraction of the photon population that is coincident with the total spin of three units or greater. Since the poles of the graviton multipole are separated by less than the Planck length, creation by the interaction of photons from a laser that are not position entangled at the same point is unlikely, though one additional spin-aligned photon must be supplied from a different direction, in order to reduce overall net momentum from  $E = pc$ . Ideally, a laser should have its entire emitted photon population in sets of three photons entangled by stimulated emission. A scalar entangled grouping cannot be produced this way due to spin angular momentum conservation, but higher spin moment groupings should also contribute, so long as their electromagnetic fields are effectively externally screened by the Planck length.

There would be almost total electromagnetic screening, though such exactly coincident photons would couple to virtual graviton pairs with the coupling constant of elementary charge leptons. Therefore, the dipole is not likely to be directly detected, but the energy loss from its production could be. The charges would have higher tendency to separate in the presence of an electric field directed parallel to the dipole, and perpendicular to the laser beam, such as could be applied by the presence of a nucleus, with which an additional aligned unit of spin must be exchanged. A graviton dipole could annihilate to produce photons again, as explained above. They should tend to recombine into four photons scattering with a spread of angles. Energy and momentum conserving breakdown products appear to be relatively degenerate, so photons produced by annihilation could be randomly polychromatic. The chance of collision by the fourth photon is higher at greater photon energies due to the reduced de Broglie wavelengths, though it should be possible to achieve pair production with lasers with photon energies less than the masses of lepton pairs. If massless charged gravitons exist, pair production should be possible to the limit of no exchange with a nucleus or applied field, with only four coincident spin-aligned photons, though not necessarily with great frequency.

At high energies, if electron-positron pair production cross section goes like

$$\sigma \propto Z^2 \log(k_0 - k_{crit}) \quad (17)$$

with  $Z$  being the charge of the nucleus,  $k_0$  being the incident photon wavenumber, and  $k_{crit}$  being the critical wavenumber for electron-positron pair production, at minimum sufficient to provide the mass of two electrons, then graviton pair production should go like

$$\sigma \propto \left(\frac{Z}{2}\right)^2 \log(k_0). \quad (18)$$

We assume here that the entirety of the laser is in spin triplet sets of photons, and that the spins of the charged nucleons are random. With ideal laser population statistics, high energy production of electron-positron pairs limits to a factor of 4 greater than graviton pairs. It is possible to approach a graviton cross section approximately equal to electron-positron cross section if all laser photons come in spin-aligned triplets and if charged nucleon spins are aligned with these triplets. The laser could be passed through a polarizer, and the nucleons could be magnetized. We see that, except under strictly



ideal conditions, graviton pair production is significantly less than electron-positron pair production at high energies. At low energies, the photoelectric effect and Compton scattering appear to usually dominate. Below the threshold of the photoelectric effect, the impinging photon de Broglie wavelengths are large, and the chance of interaction is therefore low. In general, there might be no regime, or a very limited regime, where graviton pair production is expected to both occur at detectable levels and be the dominant mode of interaction. Detection might require a combination of careful experimental tuning and precise subtraction of these background processes.

## 5. Conclusion

Our scalar gravity wave augmentation leads to an infinite family of linearly superposable radiating vacuum solutions for the Reissner-Nordström and Schwarzschild metrics. If a physical particle exists that can mediate this wave, the only plausible candidate is direct emission of entangled gravitons with opposite spin. Our model implies that gravitons carry  $\pm 1$  and 0 fundamental units of charge. The model also suggests a net emission on order of  $E_P/(2r_s)$  from black holes, in addition to Hawking radiation, before the background temperature of these standing gravity waves is considered. Our vacuum solutions imply electric and magnetic scalar quadrupoles, of anti-aligned spin 2 gravitons. Such charged gravitons could be produced as pairs of from four or more coincident, spin-aligned photons of any energy, such as could be produced via stimulated emission, though detection might require careful tuning and background subtraction. Barring the existence of charged gravitons, our treatment of the Schwarzschild metric still implies gravitational mass monopole coupling. The existence of gravitational monopole interaction solutions, to order-of-magnitude, could explain dark energy and dark matter, while being compatible with current astrophysical observations, without admitting a tunable parameter that isn't self-consistently fixed. These particles could act as an inflaton. The zero kinetic energy, zero separation limit of hypothetical charged gravitons could give us a mechanism whereby charge arises from vacuum without infinite self energy, as well as set the absolute zero energy gauge of gravity. This points toward a unification of gravity with the other fundamental forces. The quantization procedure, decomposition in basis states, and mechanical behavior, for these vacuum systems of real gravitons, is obvious nearly to the point of triviality. The mere existence of solutions for entangled, anti-spin-aligned graviton states, carrying monopole interactions in every conserved quantity, offers tremendous economy of theory, in agreement with existing observational data, without "new physics."

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